Déterminer les éventuelles asymptotes des fonctions suivantes

- 1) $f(x) = \sqrt{x^2 - 3x + 2}$
- 2) $f(x) = \dfrac{\sqrt{x^2 - 1}}{3 - x}$
- 3) $f(x) = \sqrt{x - 1} - \sqrt{x}$
- 4) $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$
- 5) $f(x) = \sqrt{\dfrac{x - 3}{x - 4}}$
- 6) $f(x) = \dfrac{\sqrt{x + 7} - 3}{x - 2}$
- 7) $f(x) = x + \sqrt{x + 5}$
- 8) $f(x) = \sqrt{x^2 + 1} - x$
- 9) $f(x) = \dfrac{x\,|x - 1|}{x^2 - 1}$
- 10) $f(x) = \dfrac{|x^2 - x - 2|}{2x + 1}$

Solutions détaillées (Claroline)

- 1) $f(x) = \sqrt{x^2 - 3x + 2}$

Dom f = $\longleftarrow; 1] \cup [2; \longrightarrow$

$\lim_{x \to 1} \sqrt{x^2 - 3x + 2} = 0$

$\lim_{x \to 2} \sqrt{x^2 - 3x + 2} = 0$

$$\lim_{x \to +\infty} \sqrt{x^2 - 3x + 2} = \lim_{x \to +\infty} \sqrt{x^2} = \lim_{x \to +\infty} x = \lim_{x \to +\infty} +\infty$$

$$\lim_{x \to -\infty} \sqrt{x^2 - 3x + 2} = \lim_{x \to -\infty} \sqrt{x^2} = \lim_{x \to -\infty} -x = \lim_{x \to -\infty} +\infty$$

$$a = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\sqrt{x^2 - 3x + 2}}{x} = \lim_{x \to +\infty} \frac{\sqrt{x^2}}{x} = \lim_{x \to +\infty} \frac{x}{x} = \lim_{x \to +\infty} 1 = 1$$

$$b = \lim_{x \to +\infty} f(x) - a\,x = \lim_{x \to +\infty} \sqrt{x^2 - 3x + 2} - x$$

$$\lim_{x \to +\infty} \sqrt{x^2 - 3x + 2} - (x) = [+\infty - \infty]$$

$$= \lim_{x \to +\infty} \frac{\sqrt{x^2 - 3x + 2} - (x)\ \sqrt{x^2 - 3x + 2} + (x)}{\sqrt{x^2 - 3x + 2} + (x)}$$

$$= \lim_{x \to +\infty} \frac{(x^2 - 3x + 2) - (x^2)}{\sqrt{x^2 - 3x + 2} + (x)} = \lim_{x \to +\infty} \frac{2 - 3x}{\sqrt{x^2 - 3x + 2} + (x)}$$

$$= \lim_{x \to +\infty} \frac{-3x}{x + \sqrt{x^2}} = \lim_{x \to +\infty} \frac{-3x}{2x} = -\frac{3}{2}$$

AO $\equiv y = x - \frac{3}{2}$ à droite

$$a = \lim_{x \to -\infty} \frac{f(x)}{x} =$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 - 3x + 2}}{x} = \lim_{x \to -\infty} \frac{\sqrt{x^2}}{x} = \lim_{x \to -\infty} \frac{-x}{x} = \lim_{x \to -\infty} -1 = -1$$

$$b = \lim_{x \to -\infty} f(x) - a\,x = \lim_{x \to -\infty} \sqrt{x^2 - 3x + 2} + x$$

$$\lim_{x \to -\infty} \sqrt{x^2 - 3x + 2} + x = [+\infty - \infty]$$

$$= \lim_{x \to -\infty} \frac{(\sqrt{x^2 - 3x + 2} + (x))\ (\sqrt{x^2 - 3x + 2} - (x))}{\sqrt{x^2 - 3x + 2} - (x)}$$

$$= \lim_{x \to -\infty} \frac{(x^2 - 3x + 2) - (x^2)}{\sqrt{x^2 - 3x + 2} - (x)} = \lim_{x \to -\infty} \frac{2 - 3x}{\sqrt{x^2 - 3x + 2} - (x)}$$

$$= \lim_{x \to -\infty} \frac{-3x}{\sqrt{x^2} - x} = \lim_{x \to -\infty} \frac{-3x}{-2x} = \frac{3}{2}$$

AO $\equiv y = \frac{3}{2} - x$ à gauche

- 2) $f(x) = \dfrac{\sqrt{x^2 - 1}}{3 - x}$

Dom f = $\longleftarrow; -1] \cup [1; 3[\cup]3; \longrightarrow$

$$\lim_{x\to -1} \frac{\sqrt{x^2-1}}{3-x} = 0$$

$$\lim_{x\to 1} \frac{\sqrt{x^2-1}}{3-x} = 0$$

$$\lim_{x\to 3} \frac{\sqrt{x^2-1}}{3-x} = [\frac{2\sqrt{2}}{0}]$$

$$\begin{cases} \lim\limits_{\substack{x\to 3 \\ <}} \frac{\sqrt{x^2-1}}{3-x} = +\infty \\ \lim\limits_{\substack{x\to 3 \\ >}} \frac{\sqrt{x^2-1}}{3-x} = -\infty \end{cases}$$

AV $\equiv x = 3$

$$\lim_{x\to +\infty} \frac{\sqrt{x^2-1}}{3-x} = \lim_{x\to +\infty} \frac{\sqrt{x^2}}{-x} = \lim_{x\to +\infty} \frac{x}{-x} = \lim_{x\to +\infty} -1 =-1$$

AH $\equiv y = -1$ à droite

$$\lim_{x\to -\infty} \frac{\sqrt{x^2-1}}{3-x} = \lim_{x\to -\infty} \frac{\sqrt{x^2}}{-x} = \lim_{x\to -\infty} \frac{-x}{-x} = \lim_{x\to -\infty} 1 =1$$

AH $\equiv y = 1$ à gauche

- 3) $f(x) = \sqrt{x-1} - \sqrt{x}$

Dom f = [1; ⟶

$$\lim_{x\to 1} \sqrt{x-1} - \sqrt{x} = -1$$

$$\lim_{x\to +\infty} \sqrt{x-1} - \sqrt{x} = [(\infty)-(\infty)]$$

$$= \lim_{x\to +\infty} \frac{\left(\sqrt{x-1} - \sqrt{x}\right)\left(\sqrt{x-1} + \sqrt{x}\right)}{\sqrt{x-1} + \sqrt{x}}$$

$$= \lim_{x\to +\infty} \frac{(x-1)^2 - x^2}{\sqrt{x-1} + \sqrt{x}} = \lim_{x\to +\infty} \frac{-1}{\sqrt{x-1} + \sqrt{x}}$$

$$= \lim_{x\to +\infty} \frac{-1}{2\sqrt{x}} = 0$$

AH $\equiv y = 0$ à droite

$$\lim_{x\to -\infty} \frac{\sqrt{x-1} - \sqrt{x}}{x} = \nexists \qquad \text{(voir domaine !)}$$

- 4) $f(x) = \dfrac{1}{\sqrt{4-x^2}}$

Dom f =]−2; 2[

$$\lim_{x\to -2} \frac{1}{\sqrt{4-x^2}} = [\frac{1}{0}]$$

$$\lim_{\substack{x\to -2 \\ >}} \frac{1}{\sqrt{4-x^2}} = +\infty$$

AV $\equiv x = -2$ à droite

$$\lim_{x\to 2} \frac{1}{\sqrt{4-x^2}} = [\frac{1}{0}]$$

$$\lim_{\substack{x\to 2\\<}} \frac{1}{\sqrt{4-x^2}} = +\infty$$

AV $\equiv x = 2$ à gauche

$$\lim_{x\to +\infty} \frac{1}{\sqrt{4-x^2}} = \nexists$$

- 5) $f(x) = \sqrt{\frac{x-3}{x-4}}$

Dom f = $\longleftarrow; 3] \cup]4; \longrightarrow$

$$\lim_{x\to 3} \sqrt{\frac{x-3}{x-4}} = 0$$

$$\lim_{\substack{x\to 4\\>}} \sqrt{\frac{x-3}{x-4}} = +\infty$$

AV $\equiv x = 4$ à droite

$$\lim_{x\to -\infty} \sqrt{\frac{x-3}{x-4}} = \lim_{x\to -\infty} \sqrt{\frac{x-3}{x-4}} = 1$$

AH $\equiv y = 1$

- 6) $f(x) = \frac{\sqrt{x+7} - 3}{x-2}$

Dom f = $[-7; 2[\cup]2; \longrightarrow$

$$\lim_{x\to -7} \frac{\sqrt{x+7} - 3}{x-2} = \frac{1}{3}$$

$$\lim_{x\to 2} \frac{\sqrt{x+7} - 3}{x-2} = [\frac{0}{0}]$$

$$= \lim_{x\to 2} \frac{\left(\sqrt{x+7} - 3\right)\left(\sqrt{x+7} + 3\right)}{(x-2)\left(\sqrt{x+7} + 3\right)}$$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)\left(\sqrt{x+7} + 3\right)} = \lim_{x\to 2} \frac{1}{\sqrt{x+7} + 3} = \frac{1}{6}$$

$$\lim_{x\to +\infty} \frac{\sqrt{x+7} - 3}{x-2} = \lim_{x\to +\infty} \frac{\sqrt{x}}{x} = \lim_{x\to +\infty} \frac{1}{\sqrt{x}} = 0$$

AH $\equiv y = 0$ à droite

$$\lim_{x\to -\infty} \frac{\sqrt{x+7} - 3}{x-2} = \nexists$$

- 7) $f(x) = x + \sqrt{x+5}$

Dom f = $[-5; \longrightarrow$

$$\lim_{x\to -5} x + \sqrt{x+5} = -5$$

$\lim_{x\to+\infty} x+\sqrt{x+5} = \lim_{x\to+\infty} x = +\infty$

pas d'AH

AO ?

$a = \lim_{x\to+\infty} \frac{f(x)}{x} =$

$\lim_{x\to+\infty} \frac{x+\sqrt{x+5}}{x} = \lim_{x\to+\infty} \frac{x}{x} = \lim_{x\to+\infty} 1 = 1$

$b = \lim_{x\to+\infty} f(x) - a\,x =$

$\lim_{x\to+\infty} \sqrt{x+5} = \lim_{x\to+\infty} \sqrt{x} = +\infty$

pas d'AO

$\lim_{x\to-\infty} x+\sqrt{x+5} = \nexists$

- 8) $f(x) = \sqrt{x^2+1} - x$

Dom f = $\mathbb{R}$

pas d'asymptote verticale

$\lim_{x\to+\infty} \sqrt{x^2+1} - x = [\infty+(-\infty)]$

$= \lim_{x\to+\infty} \frac{\left(\sqrt{x^2+1} - x\right)\left(x+\sqrt{x^2+1}\right)}{x+\sqrt{x^2+1}}$

$= \lim_{x\to+\infty} \frac{1}{x+\sqrt{x^2+1}} = \lim_{x\to+\infty} \frac{1}{x+\sqrt{x^2}}$

$= \lim_{x\to+\infty} \frac{1}{x+|x|} = \lim_{x\to+\infty} \frac{1}{2\,x} = 0$

AH $\equiv y = 0$ à droite

$\lim_{x\to-\infty} \sqrt{x^2+1} - x = [\infty+(\infty)] = +\infty$ pas d'AV à gauche

AO ?

$a = \lim_{x\to-\infty} \frac{f(x)}{x} =$

$\lim_{x\to-\infty} \frac{\sqrt{x^2+1} - x}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2} - x}{x} = \lim_{x\to-\infty} \frac{-2\,x}{x} = \lim_{x\to-\infty} -2 =-2$

$b = \lim_{x\to-\infty} f(x) - a\,x =$

$\lim_{x\to-\infty} \sqrt{x^2+1} + x = [\infty+(-\infty)]$

$= \lim_{x\to-\infty} \frac{\left(\sqrt{x^2+1} - x\right)\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1} - x} = \lim_{x\to-\infty} \frac{1}{\sqrt{x^2+1} - x}$

$= \lim_{x\to-\infty} \frac{1}{\sqrt{x^2} - x} = \lim_{x\to-\infty} \frac{1}{|x| - x}$

$= \lim_{x\to-\infty} \frac{1}{-2\,x} = 0$

AO ≡ $y = -2x$ à gauche

- 9) $f(x) = \dfrac{x|x-1|}{x^2-1}$

Dom f = $\mathbb{R} \setminus \{-1,1\}$

$\lim_{x\to-1} \dfrac{x|x-1|}{x^2-1} = [\dfrac{-2}{0}]$

$$\begin{cases} \lim_{\substack{x\to-1\\<}} \frac{x|x-1|}{x^2-1} = -\infty \\ \lim_{\substack{x\to-1\\>}} \frac{x|x-1|}{x^2-1} = +\infty \end{cases} \qquad \text{AV} \equiv x = -1$$

$\lim_{x\to1} \dfrac{x|x-1|}{x^2-1} = [\dfrac{0}{0}]$

$$\begin{cases} \lim_{\substack{x\to1\\<}} \frac{x|x-1|}{x^2-1} = -\frac{1}{2} \\ \lim_{\substack{x\to1\\>}} \frac{x|x-1|}{x^2-1} = \frac{1}{2} \end{cases}$$

$\lim_{x\to+\infty} \dfrac{x|x-1|}{x^2-1} = \lim_{x\to+\infty} \dfrac{x|x-1|}{x^2} = \lim_{x\to+\infty} \dfrac{|x-1|}{x} = 1$

AH ≡ $y = 1$ à droite

$\lim_{x\to-\infty} \dfrac{x|x-1|}{x^2-1} = \lim_{x\to-\infty} \dfrac{x|x-1|}{x^2} = \lim_{x\to-\infty} \dfrac{1}{x} - 1 = -1$

AH ≡ $y = -1$ à gauche

- 10) $f(x) = \dfrac{|x^2-x-2|}{2x+1}$

Dom f = $\mathbb{R} \setminus \{-\dfrac{1}{2}\}$

$\lim_{x\to-\frac{1}{2}} \dfrac{|x^2-x-2|}{2x+1} = [\dfrac{\frac{5}{4}}{0}] = \pm\infty$

$$\begin{cases} \lim_{\substack{x\to-\frac{1}{2}\\<}} \frac{|x^2-x-2|}{2x+1} = -\infty \\ \lim_{\substack{x\to-\frac{1}{2}\\>}} \frac{|x^2-x-2|}{2x+1} = +\infty \end{cases} \qquad \text{AV} \equiv x = -\frac{1}{2}$$

$\lim_{x\to+\infty} \dfrac{|x^2-x-2|}{2x+1} = \lim_{x\to+\infty} \dfrac{x^2-x-2}{2x+1} = \lim_{x\to+\infty} \dfrac{x^2}{2x} = \lim_{x\to+\infty} \dfrac{x}{2} = +\infty$

pas d'AH

AO ?

$a = \lim_{x\to+\infty} \dfrac{f(x)}{x} =$

$\lim_{x\to+\infty} \dfrac{|x^2-x-2|}{x(2x+1)} = \lim_{x\to+\infty} \dfrac{x^2-x-2}{x(2x+1)} = \lim_{x\to+\infty} \dfrac{x^2}{2x^2} = \dfrac{1}{2}$

$b = \lim_{x\to+\infty} f(x) - ax =$

$$\lim_{x\to+\infty} \frac{|x^2-x-2|}{2x+1} - \frac{x}{2} = \lim_{x\to+\infty} \frac{x^2-x-2}{2x+1} - \frac{x}{2} = \lim_{x\to+\infty} \frac{x^2-x-2}{2x+1} - \frac{x}{2} = \lim_{x\to+\infty} -\frac{3x+4}{4x+2} = -\frac{3}{4}$$

$$\lim_{x\to-\infty} \frac{|x^2-x-2|}{2x+1} = \lim_{x\to-\infty} \frac{x^2-x-2}{2x+1} = \lim_{x\to-\infty} \frac{x^2}{2x} = \lim_{x\to-\infty} \frac{x}{2} = -\infty$$

pas d'AH

AO ?

$$a = \lim_{x\to-\infty} \frac{f(x)}{x} =$$

$$\lim_{x\to+\infty} \frac{|x^2-x-2|}{x(2x+1)} = \lim_{x\to-\infty} \frac{x^2-x-2}{2x^2+x} = \lim_{x\to-\infty} \frac{x^2}{2x^2} = \frac{1}{2}$$

$$b = \lim_{x\to-\infty} f(x) - a\,x =$$

$$\lim_{x\to-\infty} \frac{|x^2-x-2|}{2x+1} - \frac{x}{2} = \lim_{x\to-\infty} \frac{x^2-x-2}{2x+1} - \frac{x}{2} = \lim_{x\to-\infty} -\frac{3x+4}{4x+2} = \lim_{x\to-\infty} -\frac{3}{4}$$

$$\text{AO} \equiv y = \frac{x}{2} - \frac{3}{4}$$