Domaine de définition - dérivée

- Calculer le domaine de définition et la fonction dérivée

a) $f(x) = \text{Arctg}\left(\frac{1}{x}\right)$

b) $f(x) = \text{Arccos}\left(\frac{x}{x-2}\right)$

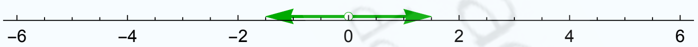
c) $f(x) = \text{Arctg}\left(\sqrt{\frac{x}{x^2-1}}\right)$

d) $f(x) = \sqrt{\text{Arctg}(x)}$

e) $f(x) = \frac{1}{\text{Arccos}(x)}$

Solutions

a) $x \neq 0 \Longleftrightarrow x \in \mathbb{R}_0$



dom $f = \mathbb{R} \setminus \{0\}$

$$f'(x) = \frac{1}{1+\frac{1}{x^2}}\left(\frac{1}{x}\right)' = 1 + \frac{1}{x^2}\left(-\frac{1}{x^2}\right) = \frac{-\frac{1}{x^2}}{1+\frac{1}{x^2}} = -\frac{1}{x^2+1}$$

b) $x - 2 \neq 0 \Leftrightarrow x \neq 2 \Leftrightarrow x \in \mathbb{R} \setminus \{2\}$

$-1 \leq \frac{x}{x-2} \leq 1$

$-1 \leq \frac{x}{x-2}$ et $\frac{x}{x-2} \leq 1$

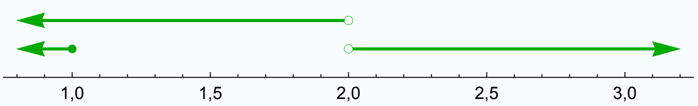
$-\frac{x}{x-2} - 1 \leq 0$ et $\frac{x}{x-2} - 1 \leq 0$

$-\frac{2(x-1)}{x-2} \leq 0$ et $\frac{2}{x-2} \leq 0$

x		1		2	
$-\frac{2(x-1)}{x-2}$	−	0	+	‖	−

x		2	
$\frac{2}{x-2}$	−	‖	+

$(\longleftarrow; 1] \cup]2; \longrightarrow) \cap (\longleftarrow; 2[) = \longleftarrow; 1]$

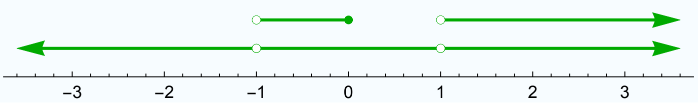


dom $f = \longleftarrow; 1]$

c) $x^2 - 1 \neq 0 \Leftrightarrow x \neq -1 \wedge x \neq 1 \Leftrightarrow x \in \mathbb{R} \setminus \{-1,1\}$

$\frac{x}{x^2-1} \geq 0 \Leftrightarrow -1 < x \leq 0 \vee x > 1 \Leftrightarrow x \in]-1; 0] \cup]1; \longrightarrow$

x		−1		0		1	
x	−	−	−	0	+	+	+
x^2-1	+	0	−	−	−	0	+
$\frac{x}{x^2-1}$	−	‖	+	0	−	‖	+



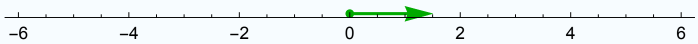
dom $f =]-1; 0] \cup]1; \longrightarrow$

$$f'(x) = \frac{1}{1+\frac{x}{x^2-1}}\left(\sqrt{\frac{x}{x^2-1}}\right)' = \frac{1}{\frac{x^2-1+x}{x^2-1}}\,\frac{1}{2\sqrt{\frac{x}{x^2-1}}}\left(\frac{x}{x^2-1}\right)'$$

$$= \frac{1}{\frac{x^2+x-1}{x^2-1}}\,\frac{1}{2\sqrt{\frac{x}{x^2-1}}}\,\frac{x^2-1-2x^2}{(x^2-1)^2} = \frac{1}{x^2+x-1}\,\frac{1}{2\sqrt{\frac{x}{x^2-1}}}\,\frac{-1-x^2}{(x^2-1)}$$

$$= \frac{-x^2-1}{2\sqrt{x(x^2-1)}\,(x^2+x-1)}$$

d) $\text{Arctg}(x) \geq 0 \Leftrightarrow x \geq 0$ et $x \in [0; \longrightarrow$



dom $f = [0; \longrightarrow$

$$f'(x) = \frac{1}{2\sqrt{\text{Arctg}(x)}}\,(\text{Arctg}(x))' = \frac{1}{2\left(x^2+1\right)\sqrt{\text{Arctg}(x)}}$$

e) $\text{Arccos}(x) \neq 0 \Leftrightarrow x \neq 1$ et $-1 \leq x \leq 1$

dom $f = [-1; 1[$

$$\left(\frac{1}{\text{Arccos}(x)}\right)' = \frac{-(\text{Arccos}(x))'}{\text{Arccos}^2(x)} = \frac{\frac{1}{\sqrt{1-x^2}}}{\text{Arccos}^2(x)} = \frac{1}{\text{Arccos}^2(x)\sqrt{1-x^2}}$$