■ Etude de fonction

$f(x) = \frac{x^2 - x}{x^2 + 1}$

1. Domaine de définition

Dom f = $\mathbb{R}$

x			0		1	
$\frac{x^2-x}{x^2+1}$	|	+	0	−	0	+

2. Limites et asymptotes

$\lim_{x\to-\infty} f(x) = 1$
AH ≡ y = 1 à gauche
$\lim_{x\to\infty} f(x) = 1$
AH ≡ y = 1 à droite

3. Etude de f'

$f'(x) = \frac{x^2 + 2x - 1}{(x^2 + 1)^2}$

x			$-1-\sqrt{2}$		$-1+\sqrt{2}$	
$\frac{x^2+2x-1}{(x^2+1)^2}$	|	+	0	−	0	+

Max = (−2.41421 , 1.20711)

Min = (0.414214 , −0.207107)

4. Etude de f"

$f''(x) = -\frac{2\left(x^3 + 3x^2 - 3x - 1\right)}{(x^2 + 1)^3}$

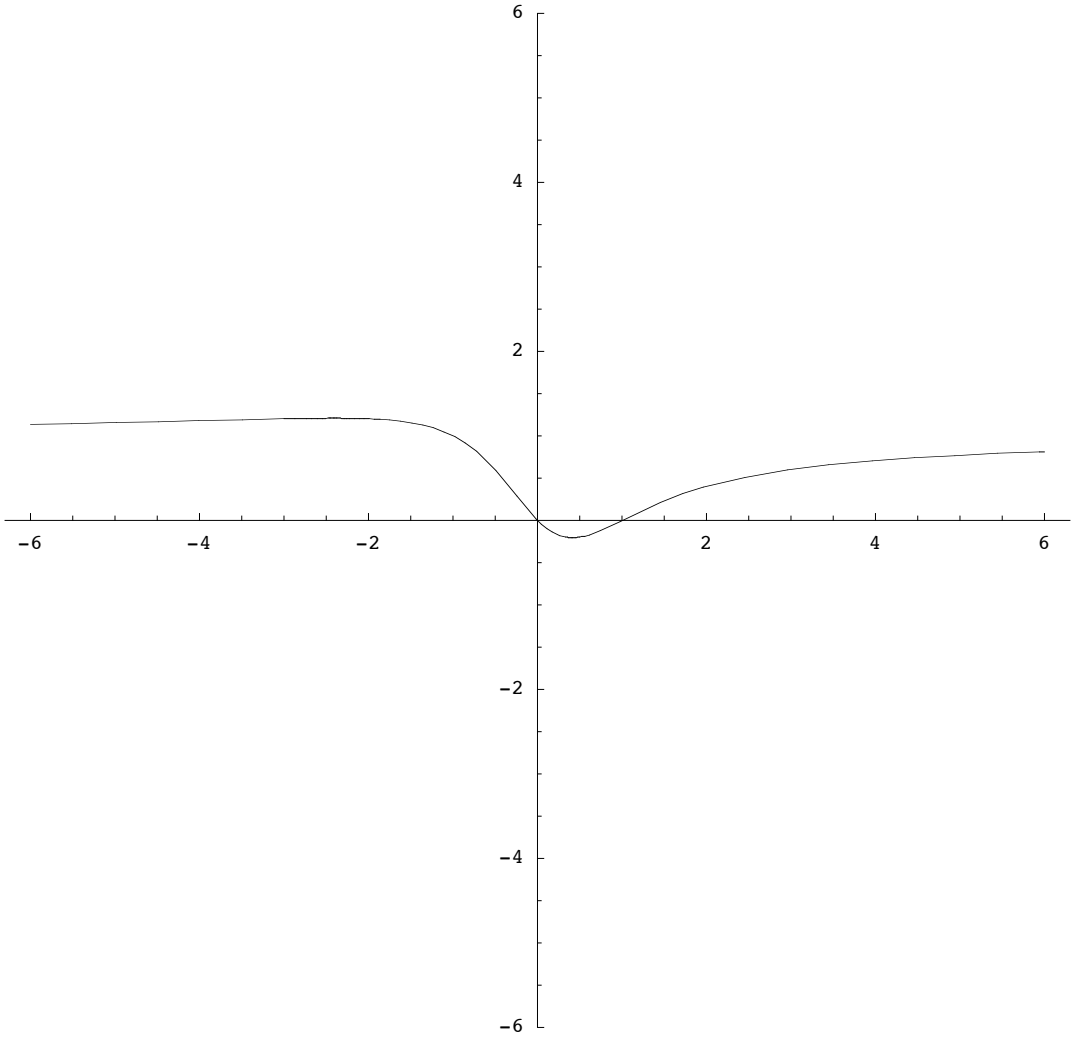
x			$-2-\sqrt{3}$		$-2+\sqrt{3}$		1	
$-\frac{2(x^3+3x^2-3x-1)}{(x^2+1)^3}$	|	+	0	−	0	+	0	−

I = (1 , 0)

I = (−3.73205 , 1.18301)

I = (−0.267949 , 0.316987)

5. Graphe de f


6
4
2
−6
−4
−2
2
4
6
−2
−4
−6