Calculer les limites aux bords du domaine de définition

$$f(x) = \frac{3x^2 - 5x + 2}{x^2 - 7x + 6}$$

Dom f = $\mathbb{R} \setminus \{1,6\}$

$$\lim_{x\to 1} \frac{3x^2 - 5x + 2}{x^2 - 7x + 6} = [\frac{0}{0}]$$

$$= \lim_{x\to 1} \frac{(x-1)(3x-2)}{(x-6)(x-1)}$$

$$= \lim_{x\to 1} \frac{3x-2}{x-6}$$

$$\lim_{x\to 1} \frac{3x-2}{x-6} = -\frac{1}{5}$$

$$\lim_{x\to 6} \frac{3x^2 - 5x + 2}{x^2 - 7x + 6} = [\frac{80}{0}]$$

x		$\frac{2}{3}$		1		6	
$3x^2 - 5x + 2$	+	0	−	0	+	+	+
$x^2 - 7x + 6$	+	+	+	0	−	0	+
$\frac{3x^2-5x+2}{x^2-7x+6}$	+	0	−	‖	−	‖	+

$$\begin{cases} \lim\limits_{\substack{x\to 6 \\ <}} \frac{3x^2-5x+2}{x^2-7x+6} = -\infty \\ \lim\limits_{\substack{x\to 6 \\ >}} \frac{3x^2-5x+2}{x^2-7x+6} = +\infty \end{cases}$$

$$\lim_{x\to +\infty} \frac{3x^2 - 5x + 2}{x^2 - 7x + 6} = \lim_{x\to +\infty} \frac{3x^2}{x^2} = \lim_{x\to +\infty} 3 = 3$$

$$\lim_{x\to -\infty} \frac{3x^2 - 5x + 2}{x^2 - 7x + 6} = \lim_{x\to -\infty} \frac{3x^2}{x^2} = \lim_{x\to -\infty} 3 = 3$$

$$f(x) = \frac{3x^2 - 4x + 1}{2x - 3}$$

Dom f = $\mathbb{R} \setminus \{\frac{3}{2}\}$

$$\lim_{x \to \frac{3}{2}} \frac{3x^2 - 4x + 1}{2x - 3} = [\frac{\frac{7}{4}}{0}]$$

x		$\frac{1}{3}$		1		$\frac{3}{2}$	
$3x^2 - 4x + 1$	+	0	−	0	+	+	+
$2x - 3$	−	−	−	−	−	0	+
$\frac{3x^2-4x+1}{2x-3}$	−	0	+	0	−	‖	+

$$\begin{cases} \lim_{x \to \frac{3}{2}, <} \frac{3x^2-4x+1}{2x-3} = -\infty \\ \lim_{x \to \frac{3}{2}, >} \frac{3x^2-4x+1}{2x-3} = +\infty \end{cases}$$

$$\lim_{x \to +\infty} \frac{3x^2 - 4x + 1}{2x - 3} = \lim_{x \to +\infty} \frac{3x^2}{2x} = \lim_{x \to +\infty} \frac{3x}{2} = +\infty$$

$$\lim_{x \to -\infty} \frac{3x^2 - 4x + 1}{2x - 3} = \lim_{x \to -\infty} \frac{3x^2}{2x} = \lim_{x \to -\infty} \frac{3x}{2} = -\infty$$

$$f(x) = \sqrt{x^2 - 4x + 7}$$

Dom f = $\mathbb{R}$

$$\lim_{x \to +\infty} \sqrt{x^2 - 4x + 7} = \lim_{x \to +\infty} \sqrt{x^2} = \lim_{x \to +\infty} x = +\infty$$

$$\lim_{x \to -\infty} \sqrt{x^2 - 4x + 7} = \lim_{x \to -\infty} \sqrt{x^2} = \lim_{x \to -\infty} -x = +\infty$$

$$f(x) = x + \sqrt{2x - 3}$$

Dom f = $[\frac{3}{2}, \longrightarrow$

$$\lim_{x \to \frac{3}{2}} x + \sqrt{2x - 3} = \frac{3}{2}$$

$$\lim_{x \to +\infty} x + \sqrt{2x - 3} = \lim_{x \to +\infty} x = +\infty$$

$$\lim_{x \to -\infty} x + \sqrt{2x - 3} \;\nexists$$

$$f(x) = \frac{\sqrt{2x+3}-2}{1-2x}$$

$$\text{Dom } f = [-\frac{3}{2}, \frac{1}{2}[\cup]\frac{1}{2}, \longrightarrow$$

$$\lim_{x\to -\frac{3}{2}} \frac{\sqrt{2x+3}-2}{1-2x} = -\frac{1}{2}$$

$$\lim_{x\to \frac{1}{2}} \frac{\sqrt{2x+3}-2}{1-2x} = [\frac{0}{0}]$$

$$= \lim_{x\to \frac{1}{2}} \frac{\left(\sqrt{2x+3}-2\right)\left(\sqrt{2x+3}+2\right)}{(1-2x)\left(\sqrt{2x+3}+2\right)}$$

$$= \lim_{x\to \frac{1}{2}} \frac{2x-1}{(1-2x)\left(\sqrt{2x+3}+2\right)}$$

$$= \lim_{x\to \frac{1}{2}} -\frac{1}{\sqrt{2x+3}+2}$$

$$\lim_{x\to \frac{1}{2}} -\frac{1}{\sqrt{2x+3}+2} = -\frac{1}{4}$$

$$\lim_{x\to +\infty} \frac{\sqrt{2x+3}-2}{1-2x} = \lim_{x\to +\infty} \frac{\sqrt{2}\sqrt{x}}{-2x} = 0$$

$$\lim_{x\to -\infty} \frac{\sqrt{2x+3}-2}{1-2x} \nexists$$