1. Trouver une équation cartésienne de la tangente à la fonction f au point d'abscisse a.

1) $f(x) = \text{Arcsin}(2x+3)$ et $a = -\dfrac{3}{2}$

2) $f(x) = 2\,\text{Arccos}\left(x^2 - \dfrac{1}{4}\right)$ et $a = \dfrac{1}{2}$

3) $f(x) = \sqrt{\text{Arcsin}(3x)}$ et $a = \dfrac{1}{6}$

4) $f(x) = -\text{Arctg}(1-x)$ et $a = 2$

5) $f(x) = \text{Arcsin}\left(\dfrac{x-1}{x+1}\right)$ et $a = \dfrac{1}{3}$

Solutions

2. Rechercher les éventuelles asymptotes de cette fonction

$f(x) = x^2\,\text{Arctg}\left(\dfrac{1}{x+1}\right)$

Solution

3. Démontrer que:

1) $\forall\ x \in [-1, 1]: \text{Arcsin}(x) + \text{Arccos}(x) = \dfrac{\pi}{2}$

2) $\forall\ x \in \mathbb{R}^{+}\backslash\{0\}: \text{Arctg}(1/x) = \text{Arccotg}\, x$

3) $\text{Arcsin}\,\dfrac{3}{5} + \text{Arcsin}\,\dfrac{4}{5} = \dfrac{\pi}{2}$

4) $\forall\ a \in \left[1, \to\right.: \text{Arcsin}\left(\dfrac{a-1}{a+1}\right) = \text{Arccos}\left(\dfrac{2\sqrt{a}}{a+1}\right)$

5) $\text{Arctg}\,\dfrac{1}{3} + \text{Arctg}\,\dfrac{1}{2} = \dfrac{\pi}{4}$

6) $\text{Arctg}\,\dfrac{1}{2} + \text{Arctg}\,\dfrac{1}{5} + \text{Arctg}\,\dfrac{1}{8} = \dfrac{\pi}{4}$

rappels:

$\forall\, x \in [-1, 1]: \sin(\text{Arcsin}(x)) = x$

$\forall\, x \in [-1, 1]: \cos(\text{Arccos}(x)) = x$

$\forall\, x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]: \text{Arcsin}(\sin(x)) = x$

$\forall\, x \in [0, \pi]: \text{Arccos}(\cos(x)) = x$

$\forall\, x \in \mathbb{R}: \text{tg}(\text{Arctg}(x)) = x$

$\forall\, x \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[: \text{Arctg}(\text{tg}(x)) = x$

Solutions

1) $\forall\ x \in [-1, 1]:$

$\text{Arcsin}(x) + \text{Arccos}(x) = \pi/2$

$\Longleftrightarrow$

$\text{Arcsin}(x) = \dfrac{\pi}{2} - \text{Arccos}(x)$

$\Longleftrightarrow$

$\sin(\text{Arcsin}(x)) = \sin\left(\dfrac{\pi}{2} - \text{Arccos}(x)\right)$

$\Longleftrightarrow$

$x = \cos(\text{Arccos}(x)) = x$

2) $\forall\ x \in \mathbb{R}^{+}\backslash\{0\}$: $\text{Arctg}\left(\dfrac{1}{x}\right) = \text{Arccotg}\, x$

$\text{Arctg}\left(\dfrac{1}{x}\right) = \text{Arccotg}(x)$

$\Longleftrightarrow$

$x = \text{cotg}\left(\text{Arctg}\left(\dfrac{1}{x}\right)\right)$

$\Longleftrightarrow$ (sachant que $\text{Arctg}\left(\dfrac{1}{x}\right) \in]0, \to$)

$x = \dfrac{1}{\text{tg}\left(\text{Arctg}\left(\frac{1}{x}\right)\right)} = 0$

3) $\text{Arcsin}\,\dfrac{3}{5} + \text{Arcsin}\,\dfrac{4}{5} = \dfrac{\pi}{2}$

$\text{Arcsin}\left(\dfrac{3}{-}\right) = \dfrac{\pi}{-} - \text{Arcsin}\left(\dfrac{4}{-}\right)$

$$\sin\left(\text{Arcsin}\left(\frac{3}{5}\right)\right) = \sin\left(\frac{\pi}{2} - \text{Arcsin}\left(\frac{4}{5}\right)\right)$$
$$\frac{3}{5} = \cos\left(\text{Arcsin}\left(\frac{4}{5}\right)\right)$$
$$\frac{3}{5} = \sqrt{1 - \sin^2\left(\text{Arcsin}\left(\frac{4}{5}\right)\right)}$$
$$\frac{3}{5} = \sqrt{1 - \left(\frac{4}{5}\right)^2}$$

4) $\forall\ a \in [1, \to\ :\ \text{Arcsin}\left(\frac{a-1}{a+1}\right) = \text{Arccos}\left(\frac{2\sqrt{a}}{a+1}\right)$

Si $a \geq 1$, $\frac{a-1}{a+1} \geq 0$ et donc $0 \leq \text{Arcsin}\left(\frac{a-1}{a+1}\right) \leq \frac{\pi}{2}$

Si $a \geq 1$, $0 < \frac{2\sqrt{a}}{a+1} \leq 1$ et donc $0 \leq \text{Arccos}\left(\frac{2\sqrt{a}}{a+1}\right) < \frac{\pi}{2}$

$$\sin\left(\text{Arcsin}\left(\frac{a-1}{a+1}\right)\right) = \sin\left(\text{Arccos}\left(\frac{2\sqrt{a}}{a+1}\right)\right)$$

$$\frac{a-1}{a+1} = \sqrt{1 - \frac{4a}{(a+1)^2}}$$

$$\frac{a-1}{a+1} = \sqrt{\frac{a^2 - 2a + 1}{(a+1)^2}}$$

5) $\text{Arctg}\left(\frac{1}{2}\right) + \text{Arctg}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

remarque: $\text{Arctg}\left(\frac{1}{2}\right)$ est compris entre 0 et $\frac{\pi}{4}$, de même pour $\frac{\pi}{4} - \text{Arctg}\left(\frac{1}{3}\right)$

$$\text{tg}\left(\text{Arctg}\left(\frac{1}{2}\right)\right) = \text{tg}\left(\frac{\pi}{4} - \text{Arctg}\left(\frac{1}{3}\right)\right)$$

on sait que $\forall\ x \in \mathbb{R} :\ \text{tg}(\text{Arctg}(x)) = x$

(par contre $\text{Arctg}\left(\text{tg}\left(\frac{3\pi}{4}\right)\right) \neq \frac{3\pi}{4}$!)

$$\frac{1}{2} = \frac{\text{tg}\left(\frac{\pi}{4}\right) - \frac{1}{3}}{\frac{1}{3}\,\text{tg}\left(\frac{\pi}{4}\right) + 1}$$

$$\frac{1}{2} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$$

6) $\text{Arctg}\left(\frac{1}{2}\right) + \text{Arctg}\left(\frac{1}{5}\right) + \text{Arctg}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

vérifier les conditions d'équivalence..

$$\text{tg}\left(\text{Arctg}\left(\frac{1}{2}\right)\right) = \text{tg}\left(\frac{\pi}{4} - \text{Arctg}\left(\frac{1}{8}\right) - \text{Arctg}\left(\frac{1}{5}\right)\right)$$

$$\frac{1}{2} = \frac{\text{tg}\left(\frac{\pi}{4}\right) - \text{tg}\left(\text{Arctg}\left(\frac{1}{8}\right) + \text{Arctg}\left(\frac{1}{5}\right)\right)}{\text{tg}\left(\frac{\pi}{4}\right)\text{tg}\left(\text{Arctg}\left(\frac{1}{8}\right) + \text{Arctg}\left(\frac{1}{5}\right)\right) + 1}$$

$$\frac{1}{2} = \frac{1 - \text{tg}\left(\text{Arctg}\left(\frac{1}{8}\right) + \text{Arctg}\left(\frac{1}{5}\right)\right)}{\text{tg}\left(\text{Arctg}\left(\frac{1}{8}\right) + \text{Arctg}\left(\frac{1}{5}\right)\right) + 1} \quad (1)$$

on a :

$$\text{tg}\left(\text{Arctg}\left(\frac{1}{8}\right) + \text{Arctg}\left(\frac{1}{5}\right)\right) = \frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{40}}$$

et donc (1) devient

$$\frac{1}{2} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$$

$$\frac{1}{2} = \frac{1}{2}$$