

$$f(x) = \operatorname{Arctg}\left(\frac{x+1}{x}\right)$$

1. Domaine de définition

Dom f = $\mathbb{R} \setminus \{0\}$

2. Signe de f

| | | | | | |
|--|---|----|---|--|---|
| x | | -1 | 0 | | |
| $\operatorname{Arctg}\left(\frac{x+1}{x}\right)$ | + | 0 | - | | + |

3. Limites et asymptotes

$$\begin{cases} \lim_{\substack{x \rightarrow 0 \\ <}} \operatorname{Arctg}\left(\frac{x+1}{x}\right) = -\frac{\pi}{2} \\ \lim_{\substack{x \rightarrow 0 \\ >}} \operatorname{Arctg}\left(\frac{x+1}{x}\right) = \frac{\pi}{2} \end{cases}$$

$$\lim_{x \rightarrow +\infty} \operatorname{Arctg}\left(\frac{x+1}{x}\right) = \frac{\pi}{4}$$

$$\lim_{x \rightarrow -\infty} \operatorname{Arctg}\left(\frac{x+1}{x}\right) = -\frac{\pi}{4}$$

$$AH \equiv \gamma = \frac{\pi}{4}$$

4. Intersection avec les axes

$$Gf \cap X = \{(-1, 0)\}$$

$$Gf \cap Y = \{\}$$

5. Etude de f'

$$f'(x) = -\frac{1}{2x^2 + 2x + 1}$$

| | | | |
|------------------------|---|---|---|
| x | | 0 | |
| $-\frac{1}{2x^2+2x+1}$ | - | | - |
| f(x) | ↗ | | ↘ |

6. Etude de f''

$$f''(x) = \frac{2(2x+1)}{(2x^2+2x+1)^2}$$

| | | | | | |
|---------------------------------|---|------------------|---|---|---|
| x | | $-\frac{1}{2}$ | | 0 | |
| $\frac{2(2x+1)}{(2x^2+2x+1)^2}$ | - | 0 | + | | + |
| f(x) | - | $-\frac{\pi}{4}$ | - | | - |

$$I : \left(-\frac{1}{2}, -\frac{\pi}{4}\right)$$

7. Tableau récapitulatif

| | | | | | | | | | |
|-----------|---------------------|---|----|---|------------------|---|----|---|---------------------|
| x | $-\infty$ | | -1 | | $-\frac{1}{2}$ | | 0 | | $+\infty$ |
| $f(x)$ | $\frac{\pi}{4}$ | + | 0 | - | $-\frac{\pi}{4}$ | - | | + | $\frac{\pi}{4}$ |
| | $y = \frac{\pi}{4}$ | | | | I | | | | $y = \frac{\pi}{4}$ |
| pente | 0 | - | -1 | - | -2 | - | -1 | - | 0 |
| concavité | 0 | - | -2 | - | 0 | + | 2 | + | 0 |

8. Graphe de f